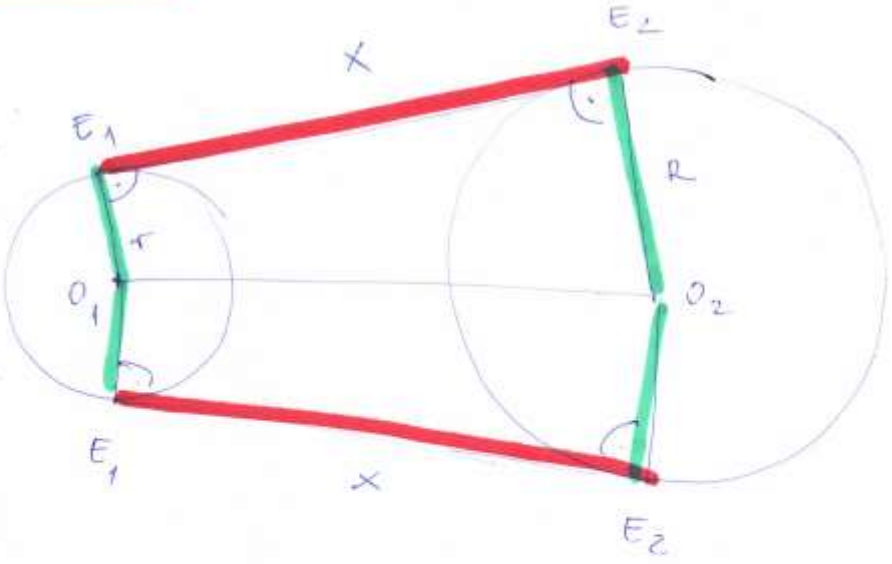
FÖLSŐ RÉSZ

1.)

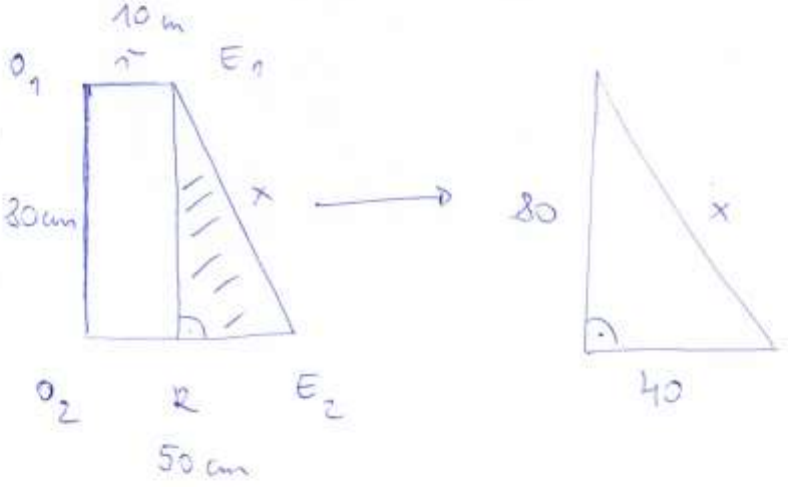


$r = 10$ cm

$R = 50$ cm

$O_1O_2 = 80$ cm

$O_1O_2E_2E_1$ trapéz, mert $O_1E_1 \parallel O_2E_2$



Pit. tétellel:

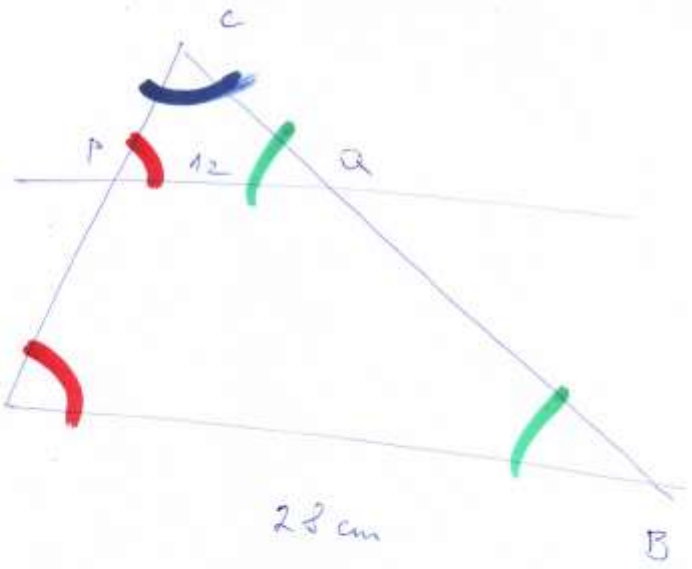
$$x = \sqrt{80^2 + 40^2} =$$

$$= \sqrt{8000} = 40\sqrt{5} \text{ cm}$$

Az érintőszakaszok hossza $40\sqrt{5}$ cm.

A feladat többi kérdésének kiszámításához szükség van a szögfüggvényekre.

2.)



$AB \parallel PQ$

$ABC\triangle \sim PQC\triangle$

(közös merőleges)

$$\Downarrow$$

$$a = \frac{AB}{PQ} = \frac{28}{12}$$

a.) $$\frac{t_{ABC}}{t_{PQC}} = a^2 \quad \rightarrow \quad a^2 = \left(\frac{28}{12}\right)^2 = \left(\frac{7}{3}\right)^2 = \frac{49}{9}$$

b.) $$t_{PQC} = \frac{9}{49} t_{ABC}$$

$$t_{ABQP} = t_{ABC} - t_{PQC} = t_{ABC} - \frac{4}{49} t_{ABC} = \frac{45}{49} t_{ABC}$$

$$\frac{t_{PQC}}{t_{ABQP}} = \frac{\frac{9}{49} t_{ABC}}{\frac{45}{49} t_{ABC}} = \frac{\frac{9}{49}}{\frac{45}{49}} = \frac{9}{49} \cdot \frac{49}{45} = \frac{9}{45}$$

3.) $\alpha : \beta : \gamma = 3 : 5 : 7$ kiegészítő $\Leftrightarrow \alpha + \gamma = \beta + \delta = 180°$

$\alpha = 3x$

$\beta = 5x$

$\gamma = 7x$

$\delta = ?$

$\alpha + \gamma = \beta + \delta$

$3x + 7x = 5x + \delta$

$10x = 5x + \delta$

$\delta = 5x$

$3x + 5x + 7x + 5x = 360°$

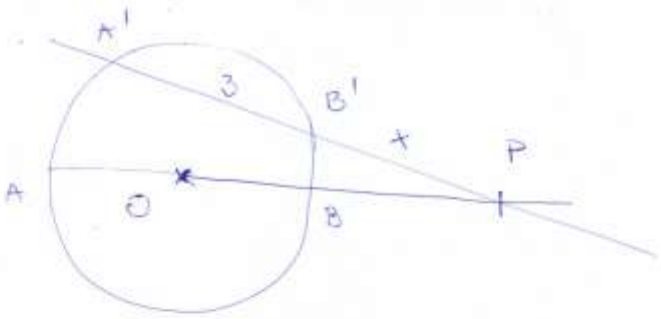
$20x = 360°$

$x = 18°$

$\pi \sim 180°$

$x = 3 \cdot 18° = 54°$	$\alpha = \frac{54°}{180°}\pi = \frac{3\pi}{10}$
$\beta = 5 \cdot 18° = 90°$	$\beta = \frac{\pi}{2}$
$\gamma = 7 \cdot 18° = 126°$	$\gamma = \frac{126°}{180°}\pi = \frac{7\pi}{10}$
$\delta = 5 \cdot 18° = 90°$	$\delta = \frac{\pi}{2}$

4.)



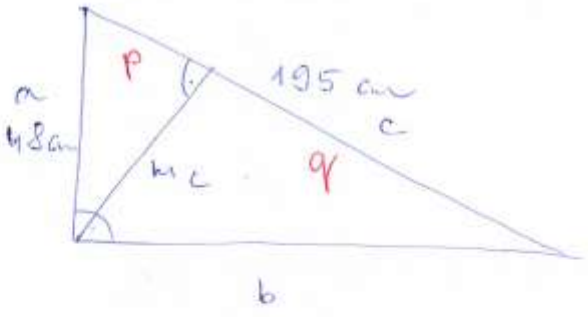
$OP = 12$ cm
$r = 6$ cm $\Rightarrow AB = 12$ cm $\Big\}$ $BP = 6$ cm
$A'B' = 3$ cm
$PB' = ?$

$x \in \mathbb{R}^+$

$$PB \cdot PA = PB' \cdot PA'$$
$$6 \cdot 18 = x \cdot (x+3)$$
$$108 = x^2 + 3x$$
$$x^2 + 3x - 108 = 0$$
$$x_{1,2} = \frac{-3 \pm \sqrt{9 + 432}}{2} = \frac{-3 \pm 21}{2} = \begin{cases} -12 \notin \mathbb{R}^+ \\ 9 \end{cases}$$

$PB' = 9$ cm $\Big\}$ hossz!
$PA' = 12$ cm

5.)



befogódtétel

$$a = \sqrt{p \cdot c}$$
$$a^2 = p \cdot c$$
$$48^2 = p \cdot 195$$
$$p = \frac{48^2}{195} \approx 11,82$$

$$q = 195 - 11,82 = 183,18$$

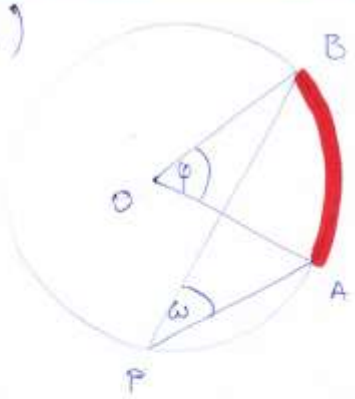
magasságtétel:

$$m_c = \sqrt{p \cdot q} = \sqrt{11,82 \cdot 183,18} = 46,53 \text{ cm}$$

$$T = \frac{c \cdot m_c}{2} = \frac{195 \cdot 46,53}{2} = 4.536,675 \text{ cm}^2$$

ALSÓ BÉR

1.)



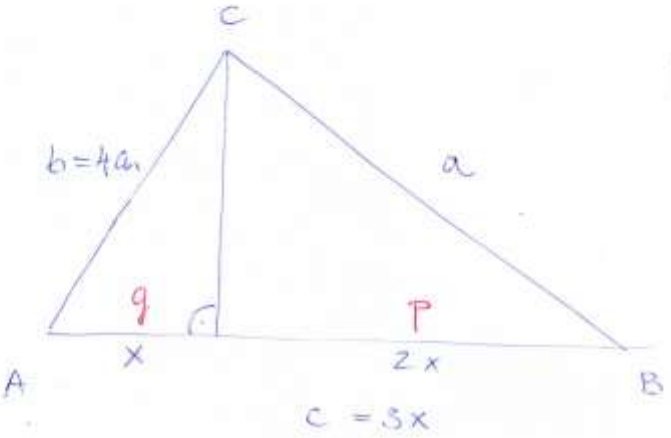
$\omega + \varphi = \frac{11\pi}{6}$ T: $\varphi = 2\omega$

$3\omega = \frac{11\pi}{6}$

$\omega = \frac{11\pi}{12}$

$\varphi = \frac{11\pi}{9}$

2.)



$c = 3x$

befogótétel alapján:

$b = \sqrt{q \cdot c} \rightarrow b^2 = q \cdot c$

$4^2 = x \cdot 3x$

$16 = 3x^2$

$x = \frac{4}{\sqrt{3}} = \frac{4\sqrt{3}}{3}$

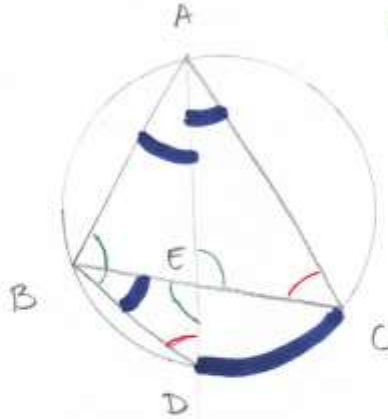
$q = \frac{4\sqrt{3}}{3}$ cm $\quad p = \frac{8\sqrt{3}}{3}$ cm

$c = 4\sqrt{3}$ cm

$a = \sqrt{p \cdot c}$

$a = \sqrt{\frac{8\sqrt{3}}{3} \cdot 4\sqrt{3}} = \sqrt{32} = 4\sqrt{2}$ cm

$a = 4\sqrt{2}$ cm



Állítás: $BD = \sqrt{AD \cdot ED}$

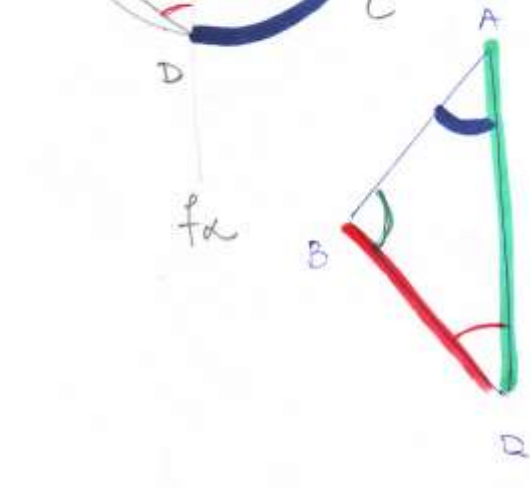
$DAC\sphericalangle = DBC\sphericalangle =$ ($\overset{\frown}{DC}$ - hez tartozó kerületi $\sphericalangle$ -ek)

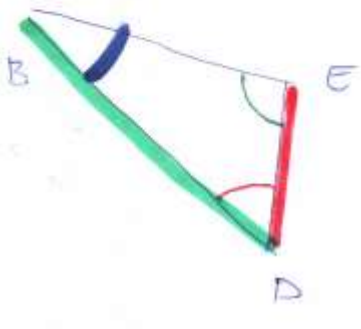
$= DAB\sphericalangle$ (közös miatt)

hasonló $\triangle$ -ek:

$DBE\triangle \sim CAE\triangle \sim DAB\triangle$

(közös megegyezik)





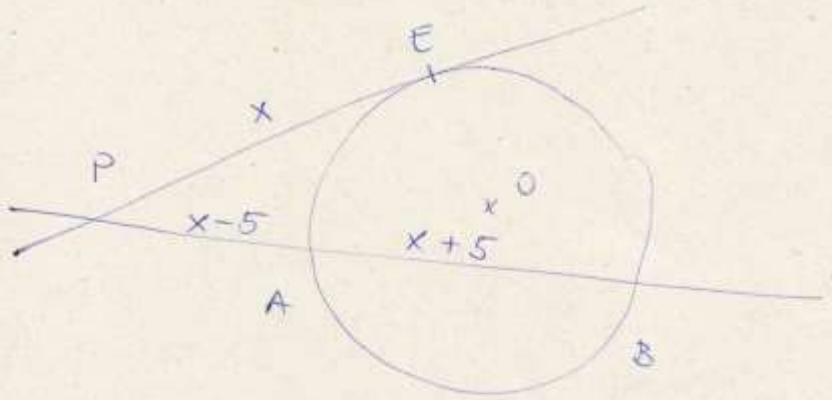
$\frac{BD}{ED} = \frac{AD}{BD}$

$BD^2 = AD \cdot ED$

$BD = \sqrt{AD \cdot ED}$

ezt kellett bizonyítani

4.)



$x \in \mathbb{R}^+$

$PE^2 = PA \cdot PB$

$x^2 = (x-5) \cdot 2x$

$x^2 = 2x^2 - 10x$

$0 = x^2 - 10x$

$0 = x(x-10)$

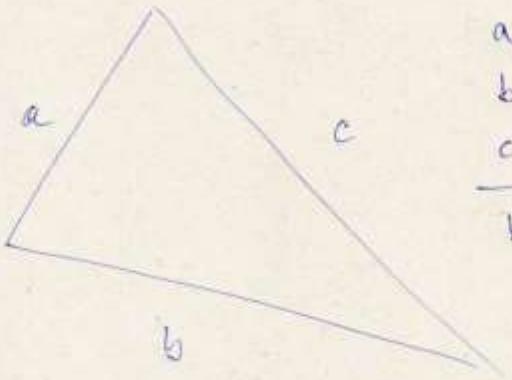
$x - 10 = 0$

$x_1 = 0 \notin \mathbb{R}^+$

$x_2 = 10$

$PE = 10$ cm

5.)



$a = 5$ cm

$b = 6$ cm

$c = 7$ cm

$k = 18$ cm

$a' = 2a$

$b' = 2b$

$c' = 2b$

$k' = 2k = 2 \cdot 18 = 27$ cm

$2 \cdot 18 = 27$

$2 = \frac{27}{18} = \frac{3}{2}$

$\frac{T'}{T} = 2^2 = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$