TRIGONOMETRIA

1. $\cos^2 x + 4\cos x = 3\sin^2 x$

$\sin^2 x + \cos^2 x = 1$ alapján

$\sin^2 x = (1 - \cos^2 x)$

behelyettesítjük

$\cos^2 x + 4\cos x = 3(1 - \cos^2 x)$

$\cos^2 x + 4\cos x = 3 - 3\cos^2 x$

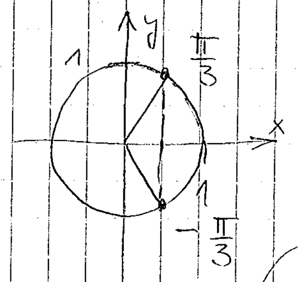
$4\cos^2 x + 4\cos x - 3 = 0$ $\qquad y := \cos x \quad y \in [-1; 1]$

$4y^2 + 4y - 3 = 0$ $\qquad a = 4 \quad b = 4 \quad c = -3$

$$y_{1,2} = \frac{-4 \pm \sqrt{16 + 48}}{8} = \frac{-4 \pm 8}{8} = \begin{cases} \frac{1}{2} \\ -\frac{12}{8} = -\frac{3}{2} \notin [-1; 1] \end{cases}$$

$y = \frac{1}{2}$

$\cos x = \frac{1}{2}$



$x_1 = \frac{\pi}{3} + k_1 \cdot 2\pi$

$x_2 = -\frac{\pi}{3} + k_2 \cdot 2\pi \quad k_{1,2} \in \mathbb{Z}$

ekv. : ekv. átalakításokat végeztem

2. a.) nem tartozik a témakörhöz

b.) $2\cos^2 x = 4 - 5\sin x$

x tehát FORGÁSSZÖG

$\sin^2 x + \cos^2 x = 1$

$\cos^2 x = (1 - \sin^2 x)$

behelyettesítjük

$2(1 - \sin^2 x) = 4 - 5\sin x$

$2 - 2\sin^2 x + 5\sin x - 4 = 0$

$-2\sin^2 x + 5\sin x - 2 = 0$

$2\sin^2 x - 5\sin x + 2 = 0$

MINDIG úgy alakítsd át, hogy olyan szögfüggvény legyen, mint amilyen az elsőfokú szögfüggvény!

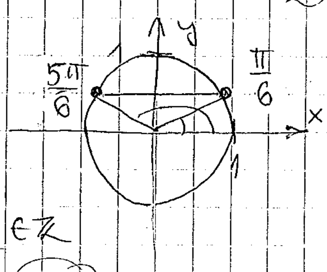
$2y^2 - 5y + 2 = 0 \qquad y := \sin x$

$a = 2 \quad b = -5 \quad c = 2 \quad y \in [-1; 1]$

$$y_{1,2} = \frac{5 \pm \sqrt{25 - 16}}{4} = \frac{5 \pm 3}{4} = \begin{cases} 2 \notin [-1; 1] \\ \frac{1}{2} \end{cases}$$

$y = \frac{1}{2}$

$\sin x = \frac{1}{2}$



$x_1 = \frac{\pi}{6} + k_1 \cdot 2\pi$

$x_2 = \frac{5\pi}{6} + k_2 \cdot 2\pi \quad k_{1,2} \in \mathbb{Z}$

ekv. ekv. átal. v.

forgásszögben

$x_1 = 30° + k_1 \cdot 360°$

$x_2 = 150° + k_2 \cdot 360°$

$k_{1,2} \in \mathbb{Z}$

3. a.) nem tartozik a témakörhöz

b.) $\sin^2 x = 2\sin x + 3$

$\sin^2 x - 2\sin x - 3 = 0$ $\quad y := \sin x \quad y \in [-1;1]$

$y^2 - 2y - 3 = 0$ $\quad a = 1 \quad b = -2 \quad c = -3$

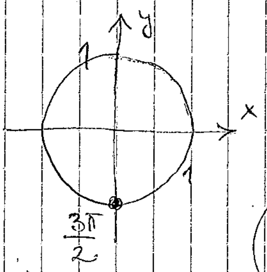
$y_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{2 \pm \sqrt{4+12}}{2} = \frac{2 \pm 4}{2} = \begin{cases} 3 \notin [-1;1] \\ -1 \end{cases}$

$y = -1$

$\sin x = -1$

$x = \frac{3\pi}{2} + k \cdot 2\pi$

$k \in \mathbb{Z}$

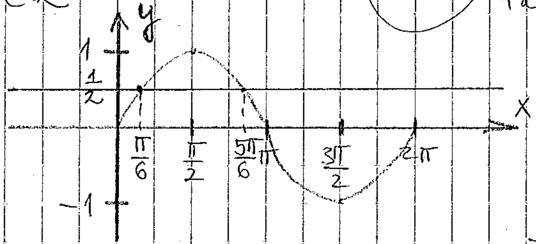


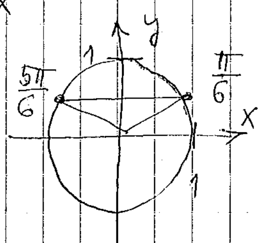
ell.: ekvivalens átalakításokat végeztem

4. $x \in [0; 2\pi]$

$\sin x = \frac{1}{2}$

$x = \left\{ \frac{\pi}{6} ; \frac{5\pi}{6} \right\}$





5.

$k(x) = \frac{5}{\cos x}$

Milyen x-re nem értelmezhető (fokokban mérve)?

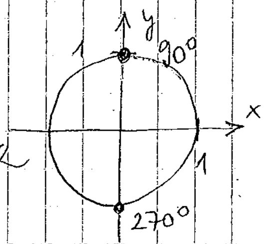
$\cos x \neq 0$, mert a tört nevezőjében van

$\cos x = 0$

$x = 90° + k \cdot 180°$

$k \in \mathbb{Z}$

esetén nem értelmezhető



6. a.) nem tartozik a témakörhöz

b.) $\sin^2\left(x - \frac{\pi}{6}\right) = \frac{1}{4}$

$\left|\sin\left(x - \frac{\pi}{6}\right)\right| = \frac{1}{2}$

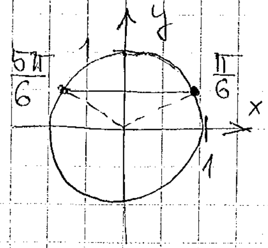
$\sin\left(x - \frac{\pi}{6}\right) = \frac{1}{2}$ $\qquad$ $\sin\left(x - \frac{\pi}{6}\right) = -\frac{1}{2}$

1. eset $\qquad$ 2. eset

6. b.) folytatás

1. eset

$\sin\left(x - \frac{\pi}{6}\right) = \frac{1}{2}$



$x - \frac{\pi}{6} = \frac{5\pi}{6} + k_1 \cdot 2\pi$

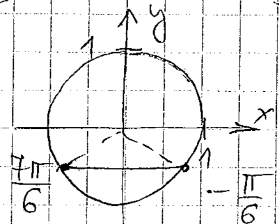
$x_1 = \pi + k_1 \cdot 2\pi$

$x - \frac{\pi}{6} = \frac{\pi}{6} + k_2 2\pi$

$x_2 = \frac{\pi}{3} + k_2 \cdot 2\pi$ $\quad k_{1,2} \in \mathbb{Z}$

2. eset

$\sin\left(x - \frac{\pi}{6}\right) = -\frac{1}{2}$



$x - \frac{\pi}{6} = \frac{7\pi}{6} + k_3 \cdot 2\pi$

$x_3 = \frac{4\pi}{3} + k_3 \cdot 2\pi$

$x - \frac{\pi}{6} = -\frac{\pi}{6} + k_4 \cdot 2\pi$

$x_4 = 0 + k_4 \cdot 2\pi$ $\quad k_{3,4} \in \mathbb{Z}$

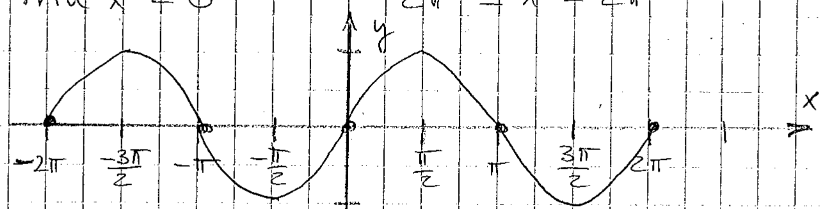
ell. ekv. átal. v.

$M: \; x = \left\{ 0 + k_4 \cdot 2\pi ;\; \frac{\pi}{3} + k_2 \cdot 2\pi ;\; \pi + k_1 \cdot 2\pi ;\; \frac{4\pi}{3} + k_3 \cdot 2\pi \right\}$ $\quad k_{1,2,3,4} \in \mathbb{Z}$

7. a) $x \mapsto \sin x \; (x \in \mathbb{R})$ periódusa: 2π IGAZ

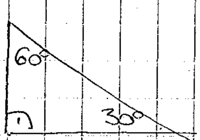
b.) $x \mapsto \sin(2x) \; (x \in \mathbb{R})$ periódusa: 2π HAMIS

8. $\sin x = 0 \qquad -2\pi \le x \le 2\pi$



zh-ek $\quad x = \{-2\pi ;\; -\pi ;\; 0 ;\; \pi ;\; 2\pi\}$

9. A.



$\sin 30° = \frac{1}{2}$ $\boxed{\text{IGAZ}}$

B. $\sin\alpha = \frac{1}{2} \Leftrightarrow \alpha = 30°$ $\quad \beta + \gamma = 150°$, nem tudjuk, hogy ebből β és γ mekkora.

$\boxed{\text{HAMIS}}$

C. b-ü △ másik két szöge hegyesszög, a hegyesszögeknek mindig van tangense

$\text{tg}\, x$ ÉT: $x \in \mathbb{R} \setminus \left\{\frac{\pi}{2} + k \cdot \pi\right\}$ $\boxed{\text{HAMIS}}$

D. $\cos x$ ÉT: $x \in \mathbb{R}$ $\boxed{\text{IGAZ}}$

bm. x-nek van koszinusza

10.

$A = \lg \frac{1}{10} = \lg 10^{-1} = -1$

$B = \cos 8\pi = \cos 2\pi = 1$

$A < B$

11. a.) nem tartozik a témakörhöz

b.) $\sin^2 x = 1 + 2\cos x$

$\sin^2 x + \cos^2 x = 1$

$\sin^2 x = 1 - \cos^2 x$

$1 - \cos^2 x = 1 + 2\cos x \quad / -1$ behelyettesítés

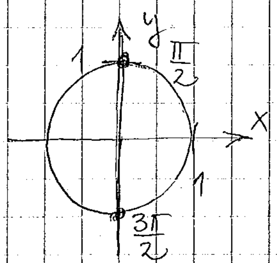
$-\cos^2 x - 2\cos x = 0 \quad / \cdot(-1)$

$\cos^2 x + 2\cos x = 0$

$\cos x(\cos x + 2) = 0$

$ab = 0 \Leftrightarrow a = 0$ vagy $b = 0$

$\cos x = 0$



$x = \frac{\pi}{2} + k \cdot \pi \quad k \in \mathbb{Z}$

$\cos x + 2 = 0$

$\cos x = -2$

$\cos x \in [-1; 1]$ miatt ennek nincs megoldása

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12. $\alpha = \frac{\pi}{4} \qquad \alpha = 45°$

13. a, b.) nem tartozik a témakörhöz

c.) $2\cos^2 x + 3\cos x - 2 = 0 \qquad x \in [-\pi; \pi]$

$2y^2 + 3y - 2 = 0 \qquad a = 2,\ b = 3,\ c = -2 \qquad y := \cos x \qquad y \in [-1; 1]$

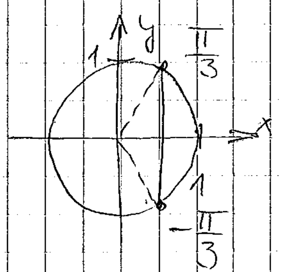
$y_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-3 \pm \sqrt{9 + 16}}{4} = \frac{-3 \pm 5}{4} = \left\langle \begin{array}{l} \frac{1}{2} \\ -2 \notin [-1; 1] \end{array} \right.$

$\cos x = \frac{1}{2}$

$x \in [-\pi; \pi]$ miatt

$x_1 = \frac{\pi}{3}$

$x_2 = -\frac{\pi}{3}$



$x = \left\{ -\frac{\pi}{3}; \frac{\pi}{3} \right\}$

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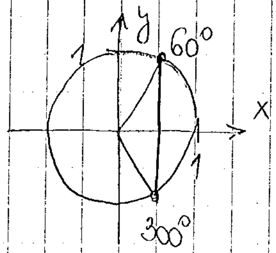
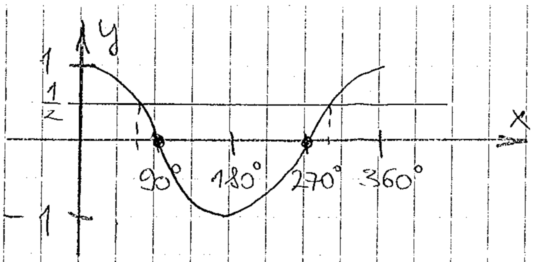
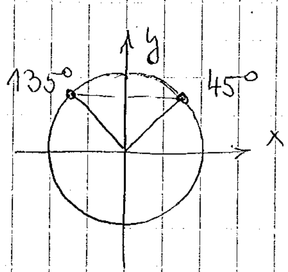
14. $\cos\alpha = \frac{1}{2}$





$\alpha = \{60°; 300°\}$

15. $\sin\alpha = \frac{\sqrt{2}}{2}$

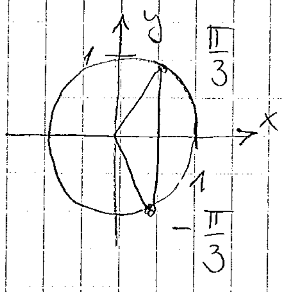


$\alpha = \{45°; 135°\}$

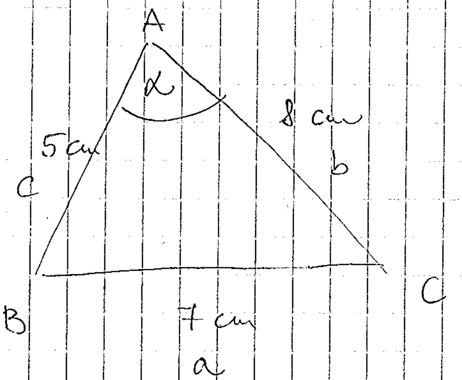
16. $x \in [-\pi; \pi]$

$\cos x = \frac{1}{2}$

$x = \{-\frac{\pi}{3}; \frac{\pi}{3}\}$



17. a.)



a koszinusz tétel alapján:

$a^2 = b^2 + c^2 - 2bc\cdot\cos\alpha$

$49 = 64 + 25 - 2\cdot 5\cdot 8\cdot\cos\alpha$

$\cos\alpha = \frac{49-(64+25)}{-80} = \frac{1}{2}$

$\cos\alpha = \frac{1}{2} \rightarrow \boxed{\alpha = 60°}$

b.) $x \in [0; 2\pi]$

$\cos^2 x = \frac{1}{4}$

$|\cos x| = \frac{1}{2}$

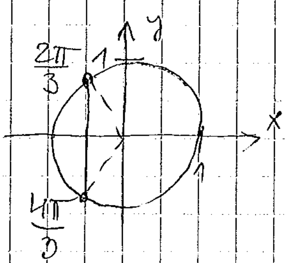
1. eset: $\cos x = \frac{1}{2}$ lásd: 16. feladat

$x_1 = \frac{\pi}{3} + k_1\cdot 2\pi$

$x_2 = -\frac{\pi}{3} + k_2\cdot 2\pi$ $k_{1,2} \in \mathbb{Z}$

2. eset:

$\cos x = -\frac{1}{2}$



$x_3 = \frac{2\pi}{3} + k_3\cdot 2\pi$

$x_4 = \frac{4\pi}{3} + k_4\cdot 2\pi$ $k_{3,4} \in \mathbb{Z}$

ell.: ekvivalens átalakításokat végeztem

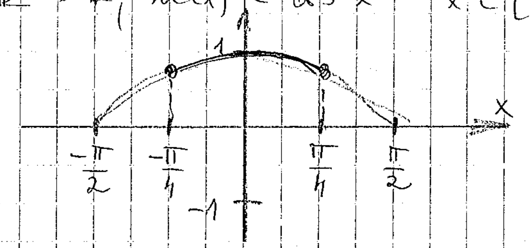
17. c.)

I. $f: \mathbb{R} \to \mathbb{R},\ f(x) = \sin x$ páratlan fv. $\boxed{\text{IGAZ}}$

II. $g: \mathbb{R} \to \mathbb{R},\ g(x) = \cos(2x)$ ÉK: $y \in [-2;2]$ $\boxed{\text{HAMIS}}$

↳ x tg mentén történő nyújtás nem befolyásolja az ÉK-t!

III. $h: \mathbb{R} \to \mathbb{R},\ h(x) = \cos x$ $x \in \left[-\frac{\pi}{4}; \frac{\pi}{4}\right]$-on szig. mon. nő

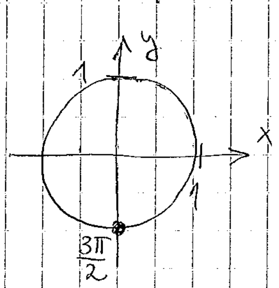


$\boxed{\text{HAMIS}}$

18. $x \in [0; 2\pi]$

$\sin x = -1$

$x = \frac{3\pi}{2}$



19. ÉK $x \mapsto 1 + \cos x$

$\cos x \in [-1; 1]$

$1 + \cos x \in [0; 2]$ ÉK: $y \in [0; 2]$